

第六届西南数论研讨会
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报告摘要

The Sixth Southwest Workshop on Number Theory
(December 13-16, 2019, Chongqing)

Abstract

1. Diophantine equations involving Euler's totient function

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Abstract. In this talk, we consider the equations involving Euler's totient function ϕ . In particular, we prove that the equation $\phi(x^m - y^m) = x^n - y^n$ has no solutions in positive integers x, y, m, n except for the trivial solutions $(x, y, m, n) = (a + 1, a, 1, 1)$, where a is a positive integer, and the equation $\phi((x^m - y^m)/(x - y)) = (x^n - y^n)/(x - y)$ has no solutions in positive integers x, y, m, n except for the trivial solutions $(x, y, m, n) = (a, b, 1, 1)$, where a, b are integers with $a > b \geq 1$. This is a joint work with Hao Tian.

2. An oscillation theorem on the additive representative function over \mathbb{N}

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Abstract. In this talk, we will introduce our recent work concerning the additive representative function over \mathbb{N} . Let A be an infinite non-empty subset of \mathbb{N} . For each $n \in \mathbb{N}$, define $r_{A,A}(n) := |\{(a, b) : a, b \in A, a + b = n\}|$ and $R_{A,A}(n) := \sum_{j \leq n} r_{A,A}(j)$. For example, we show that if the function $R_{A,A}(n)$ is well-distributed in some sense, then it can't be very well-distributed. Explicitly, if for some constant $c > 0$,

$$\limsup_{n \rightarrow \infty} \frac{|R_{A,A}(n) - cn|}{n^{\frac{1}{4}}} < +\infty$$

then for some constant $\delta > 0$, the set

$$\{n \in \mathbb{N} : |R_{A,A}(n) - cn| \geq \delta n^{\frac{1}{4}}\}$$

has a positive lower density. This result implies the well-known Erdős-Fuchs theorem. And the similar generalization for multi-sum is obtained, too. This talk is based on some work jointly with professor Hao Pan.

3. Non-vanishing theta values of characters with special prime conductors

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Abstract. Let p be a prime of the form $2\ell + 1$ or $4\ell + 1$, where ℓ is also a prime. We prove $\theta_\chi(i) \neq 0$ for all primitive Dirichlet characters χ with conductor p except for the quadratic and the quartic ones. Our results generalize a theorem of Bengoechea, which asserts $\theta(\chi, i) \neq 0$ for non-quadratic χ with “large” prime conductor $p = 2\ell + 1$, where ℓ is also a prime.

4. On the automaticity of the Hankel determinants of a family of automatic sequences

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Abstract. Hankel determinants and automatic sequences are two classical subjects widely studied in Mathematics and Theoretical Computer Science. However, these two topics were considered totally independently, until in 1998, when Allouche, Peyrière, Wen and Wen proved that all the Hankel determinants of the Thue-Morse sequence are nonzero. This property allowed Bugeaud to prove that the irrationality exponents of the Thue-Morse-Mahler numbers are exactly 2. Since then, the Hankel determinants of several other automatic sequences, in particular, the paperfolding sequence, the Stern sequence, the period-doubling sequence, are studied by Coons, Vrbik, Guo, Wu, Wen, Bugeaud, Fu, Han, Fokkink, Kraaikamp, and Shallit. On the other hand, it is known that the Hankel determinants of a rational power series are ultimately zero, and the Hankel determinants of a quadratic power series over finite fields are ultimately periodic. It is therefore natural to ask if we can obtain similar results about the Hankel determinants of algebraic series. In this talk, we provide a partial answer to this question by establishing the automaticity of the reduced Hankel determinants modulo 2 of a family of automatic sequences. As an application of our result, we give upper bounds for the irrationality exponent of a family of automatic numbers.

5. Some results on sum-product

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Abstract. The study on sum-product is very important in the additive combinatorics. In the talk, the speaker will introduce some theorems with their applications on sum-product.

6. Continued fractions: their structures and applications

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Abstract. Hurwitz continued fractions and Tasoev continued fractions are two typical regular (or simple, normal) continued fractions whose sequences of partial quotients form arithmetic series and geometric series, respectively. First, we mention their structures and expressions. Next, we find (non-regular and regular) continued fractions for infinite reciprocal sums of Fibonacci, Lucas and Tribonacci numbers. Finally, as applications, we show some relations of (non-regular and regular) continued fractions and tree graphs. In particular, Jacobsthal numbers can be interpreted.

7. 向量场与自同构群

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Abstract. 向量场与自同构群的深刻内在联系涉及几何、物理、数论等很多领域, 是自然界中的一个普遍规律, 也是一个强有力的工具. 本报告通过具体例子解释这个规律, 并说明其在数论中的表现和意义.

8. 一个类数公式

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Abstract. 我们给出一个类数公式:

$$\varepsilon_p^{-12h(p)h(-p)} = \prod_{\substack{0 \leq m \leq \frac{p-1}{2} \\ p \nmid m^2+1}} \Delta\left(\frac{m+i}{p}\right)^{\left(\frac{m^2+1}{p}\right)},$$

这里素数 $p \equiv 1 \pmod{4}$, $h(*)$ 是二次域 $Q(\sqrt{*})$ 的类数, ε_p 是实二次域 $Q(\sqrt{p})$ 的基本单位, $\left(\frac{*}{p}\right)$ 是模 p 的 Legendre 二次剩余符号, Δ 是全模群 $SL_2(\mathbb{Z})$ 的那个权为 12 的尖形式, 即所谓的判别式函数.

我们也进行一些讨论.

9. On the diophantine equation $3^x + 3^y + 2 = z^5$

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Abstract. In this talk, we prove that $(x, y, z) = (3, 1, 2)$ is the only positive integer solution of diophantine equation $3^x + 3^y + 2 = z^5$.

10. Exponential sums with multiplicative coefficients

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Abstract. In this talk, we shall introduce our series of work concerning cancellation in additively twisted sums of the form $\sum_{n \leq x} a(n)e(n\alpha)$, where $a(n)$ is a multiplicative function. In particular, we prove cancellation in these twisted sums for some multiplicative functions related to the coefficients of automorphic L -functions.

11. Neighborhood of F_p vertices in the supersingular isogeny graph

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Abstract. It is assumed to be a hard problem to compute explicitly the isogenies between supersingular elliptic curves. To attack this problem, one needs to have good understanding of the supersingular isogeny graph over a given prime field. In this talk we report our recent progress about neighborhood of the F_p vertices in this graph. This is a joint work with Songsong Li and Zheng Xu.

12. On subsequence sums of a zero-sum free sequence over finite abelian groups

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Abstract. Let G be a finite abelian group and S be a sequence with elements of G . Let $\Sigma(S) \subset G$ denote the set of group elements which can be expressed as the sum of a nonempty subsequence of S . We call S zero-sum free if $0 \notin \Sigma(S)$. In this talk, we introduce some progressions on $|\Sigma(S)|$ when S is a zero-sum free sequence satisfying $\langle S \rangle$ is not cyclic. This is a joint work with Y. Li et al.

13. The Lang-Trotter conjecture for the elliptic curve $y^2 = x^3 + Dx$

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Abstract. Let E be an elliptic curve over \mathbb{Q} . For a fixed integer r , define the prime-counting function $\pi_{E,r}(x) := \sum_{p \leq x, a_E, a_p=r} 1$. The Lang-Trotter conjecture predicts that

$$\pi_{E,r}(x) = C_{E,r} \cdot \frac{\sqrt{x}}{\log x} + o\left(\frac{\sqrt{x}}{\log x}\right)$$

as $x \rightarrow \infty$, where $C_{E,r}$ is a specific non-negative constant.

It is open whether there exists a polynomial in one variable of degree 2 that represents infinitely many primes. For example, at present, we do not know whether the polynomial $x^2 + 1$ represents infinitely many primes. The Hardy-Littlewood conjecture gives a similar asymptotic formula as above for the number of primes of the form $ax^2 + bx + c$.

We establish a relationship between the Hardy-Littlewood conjecture for $x^2 + 1$ and the Lang-Trotter conjecture for the elliptic curve $y^2 = x^3 + Dx$.

14. 三四次互反律、三四次同余式与 Lucas 序列同余式

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Abstract. 本报告介绍了古典的三四次互反律和有理三四次互反律的发展史, 概述了报告人在 17 篇有关论文中的主要结果, 特别是解决一般整数 m 以及二次域基本单位为素数 p 的三四次剩余的判别条件难题, 给出模为素数的三四次同余式解数及解的构造, 揭示四次同余式模素数 p 的不同取值个数与二次型参数及有限域上椭圆曲线点数的联系, 对 Lucas 序列 U_n 解决 $p|U((p-1)/3)$ 与 $p|U((p-1)/4)$ 的判别条件问题.

15. New series for powers of π and related congruences

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Abstract. In this talk we mainly introduce the author's new discoveries in 2019 concerning series for powers of π . We will deduce many new series for powers of π related to Ramanujan-type series for $1/\pi$, and also present various new conjectural series for $1/\pi$ involving generalized central trinomial coefficients. We also introduce our general conjectures characterizing Ramanujan-type series for $1/\pi$ via congruences.

16. The first negative Fourier coefficient of Hecke-Maass form

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Abstract. Let f be a Hecke-Maass new form for the Hecke congruence group $\Gamma_0(N)$, n_f denotes the smallest integer such that Hecke eigenvalue $\lambda_f(n_f) < 0$ and $(n_f, N) = 1$. Denote $Q_f = (3 + |\nu_f|^2)N$ the conductor of f . In this talk, it is proved that

$$n_f \ll Q_f^{\frac{1}{2} - \frac{1}{20} + \varepsilon}.$$

This is a joint work with Yuk-Kam Lau, Ming Ho Ng and Yingnan Wang.

17. An attempt to Goldbach conjecture and the geometric basis of some Siegel upper half space

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Abstract. This report is managed to demonstrate some ideas on an attempt to Goldbach conjecture from a different point of view, and to give some basic results on the attempt concerning the symplectic geometry of a related Siegel upper half space of order 2.

18. Quadratic polynomials at prime arguments

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Abstract. It is a fundamental and challenging problem to determine in general whether a given irreducible polynomial in $\mathbb{Z}[X]$ can capture infinitely many prime values. This is known in the linear case in view of Dirichlet's theorem on primes in arithmetic progressions, but no answer is valid for any non-linear cases. A much more ambitious conjecture asserts that the above infinitude also holds if one is restricted to prime variables and there are no fixed prime factors; however, even the linear case seems beyond the current approach as predicted by the twin prime conjecture. Nevertheless, we are nowadays much heartened since $p+h$ can present infinitely many primes for certain h with $1 < |h| \leq 7 \times 10^7$, thanks to Zhang's breakthrough on prime gaps.

In this talk, we are interested in the case of quadratic polynomials at prime arguments. It is of course beyond the current approach to prove the infinitude of primes captured by such polynomial, and alternatively, we consider the greatest prime factors and almost prime values as two approximations. This is a joint work with Ping Xi.

19. Three problems of Katz on Kloosterman sums

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Abstract. Nicholas Katz proposed three problems on Kloosterman sums in 1980, including their sign changes, equidistributions and modular structures. In this talk, we will discuss some recent progresses towards the three problems made by analytic number theory combining certain tools from ℓ -adic cohomology.

20. Error term of mean value for binary Egyptian fractions

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Abstract. We study the size of the error term of the mean value theorem for binary Egyptian fractions. We get an error term of prime number theorem type unconditionally. Under Riemann Hypothesis, a power saving can be obtained. The mean value in short interval is also considered. Using the method of Tong-type truncated Voronoi formula, we can get an asymptotic formula for the moment under Generalised Riemann Hypothesis.

21. On the m -th Dedekind sums

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Abstract. Let h, m, q be integers with $m, q > 0$ and $(h, q) = 1$. In this talk, we will define the m -th Dedekind sums $S(h, m, q)$ which is a generalization of the classical Dedekind sums and Cochrane sums, and study some arithmetic properties of the m -th Dedekind sums and give an asymptotic formula for mean square value of $S(h, m, q)$ by using the properties of character sums. A sharp asymptotic formula for a new mean value of Dirichlet L-functions will be also obtained.

22. 一些与abc有关的问题与猜测

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Abstract. 我们将介绍一些有关abc的一些数论问题和猜测. 特别, 我们将介绍与指数不定方程有关的猜想及相关的结论.

23. On a generalization of the Euler totient function

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Abstract. J. Kaczorowski defined the generalized Euler totient function $\varphi(n, F)$ corresponding to a polynomial Euler product F . Let $E(x, F)$ denote the error term in the asymptotic formula of the summatory function of $\varphi(n, F)$. J. Kaczorowski proved that the mean square of $E(x, F)$ has an asymptotic formula if F satisfies GRH. We shall prove a sharper asymptotic formula under GRH.

24. The generalized quadratic Gauss sums and its sixth power mean

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Abstract. The main purpose of this talk is using the elementary methods and the properties of the Legendre's symbol to study the computational problem of the sixth power mean of a certain generalized

quadratic Gauss sums, and give an exact calculating formula for it. This is a joint work with Xingxing Lv.

25. Simultaneous non-vanishing of modular L -functions on average

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Abstract. According to the grand Riemann hypothesis, the non-trivial zeros of the modular L -function associated to a Hecke eigenform of weight k lie on the vertical line with real part $k/2$. The study of zero-free regions of modular L -functions is therefore of extreme importance in analytic number theory. In this talk, via Diamantis and O'Sullivan's double Eisenstein series and its connection with products of modular L -functions, we prove a simultaneous non-vanishing result of modular L -functions on average. This is a joint work with Choie and Kohnen.

26. Slim exceptional sets: a pair of diagonal quadratic equations in eight primes

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Abstract. We shall consider the slim exceptional set on pairs of diagonal quadratic equations in 8 prime variables. Let $E(N)$ denote the cardinality of the underlying exceptional set. Subject to certain conditions on the coefficients of the equations, we prove that $E(N) \ll (\log N)^{14}$.